16. PROBABILITY

- Definition
- Sample space and sample outcome
- Event
- Types of events(impossible & sure, simple, compound)
- Algebra of events
- Mutually Exclusive events
- Exhaustive events
- Axiomatic approach to probability

Probability-Definition

- A measure of uncertainty of various phenomenon.
- Probability = $\frac{\text{No. of favorable outcome}}{\text{Total no. of outcomes}}$

Ex. Tossing a coin or getting rain.

Probability of getting a head, P(H)= $\frac{1}{2}$

Probability of getting a tail, P(T)= 1/2

Random Experiments

- Daily we perform many activities which have a fixed result no matter any number of times they are repeated.

Example

Given any triangle, without knowing the three angles, we can definitely say that the sum of measure of angles is 180°.

- We also perform many experimental activities where the result may not be same.

Example

When a coin is tossed it may turn up a head or a tail, but we are not sure which one of these results will actually be obtained.

Such experiments are called *random experiments*.

Approach to Probability: -

- **Statistical approach:** Observation and data collection.
- **Classical approach:** Only ***Equal probable*** events.
- **Axiomatic approach:** For real life events. It closely relates to set theory.

E.g. A committee of 2 person is elected from 3 men and 4 women .

What is the probability committee has no man.

Sample space & sample outcome

- **Sample space:** The set of all possible outcomes of a random experiment is called the *sample space*.
- **Sample point**: Each element of the sample space is called a sample point.

Example: Toss a coin once => Sample space= {H,T}

Example: Toss a coin twice

=> Sample space= {HH,TT,HT,TH}

Example:
A boy has a 1 rupee coin, a 2 rupee coin and a 5 rupee coin in his pocket. He takes out two coins out of his pocket, one after the other. Find the sample space.

Example:
A person is noting down the number of accidents along a busy highway during a year.

Example
A coin is tossed. If it shows head, we draw a ball from a bag consisting of 3 blue and 4 white balls; if it shows tail we throw a die. Describe the sample space of this experiment.

Example
Consider the experiment in which a coin is tossed repeatedly until a head comes up. Describe the sample space.

Event

- It is the set of favorable outcomes.
- Also, event is a subset of the sample space

Example: When a dice is thrown, what is the event of number >4 ?

S={1,2,3,4,5,6}

E={5,6}

Types of Events

- 1. Impossible and Sure event
- 2. Simple event
- 3. Compound event

- **1. Impossible and Sure event:**

***Impossible event*:** ϕ , **E.g.-** Getting a no. $x > 7$ on a dice.

***Sure event*: S** , **E.g.-** Getting a no. $0 < x < 7$ on a dice.

- **2. Simple event**
  A simple event has only one sample point of a sample space.

Example – Rolling a die. Event A is getting 3.

S = {1,2,3,4,5,6}

E_A = {3}

- **3. Compound event**
  A compound event has more than one sample point of a sample space.

Example – Rolling a die. Event A is getting a odd no.

S = {1,2,3,4,5,6}

E_A = {1,3,5}

Algebra of Events

- 1. Complimentary events
- 2.Event “ A or B “
- 3.Event “ A and B”
- 4.Event “ A but not B “

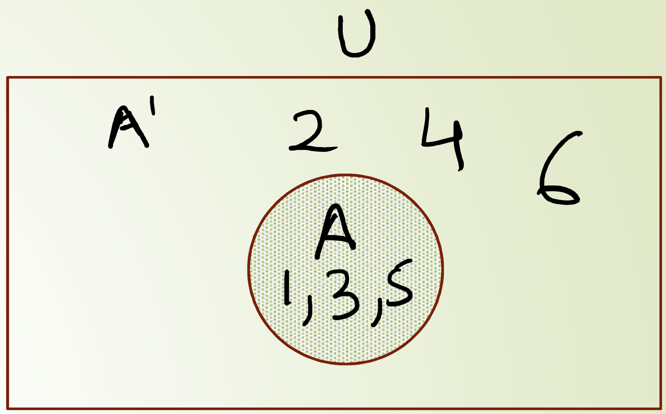
1. Complimentary event, A’

- A’ = not A

Example: A = a odd no. in die

A= { 1,3,5}

A’=U – A = {2,4,6}



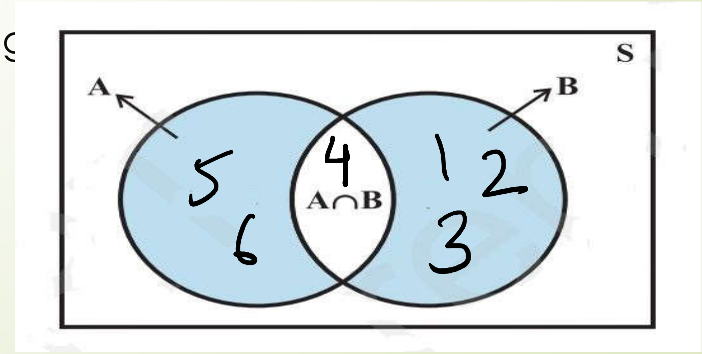
2. Event (A or B)

- A U B

Example: Event A is getting X≥4 and B is getting

A={4,5,6} and B={1,2,3,4}

A U B = {1,2,3,4,5,6}



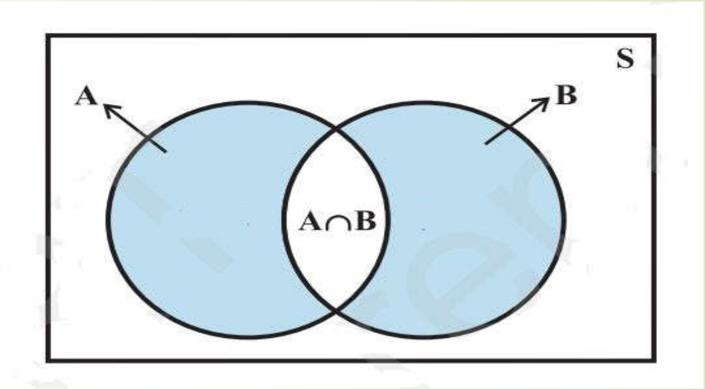
3. Event A and B

- **A ∩ B**

Example: Event A is getting X≥4 and B is getting X≤4

A={4,5,6} and B={1,2,3,4}

A ∩ B = {4}



4. Event A but not B

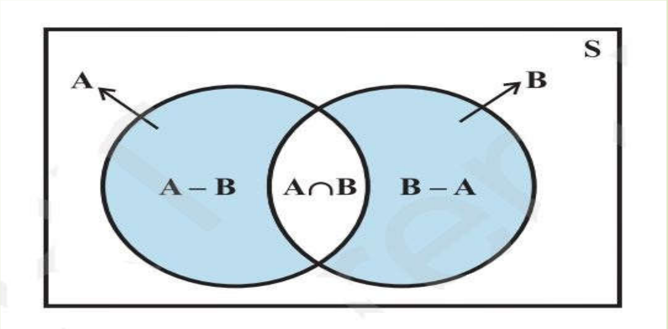
- **A – B**

Example: Event A is getting X≥4 and B is getting X≤4 .

A={4,5,6} and B={1,2,3,4}

A –B = {1,2,3}

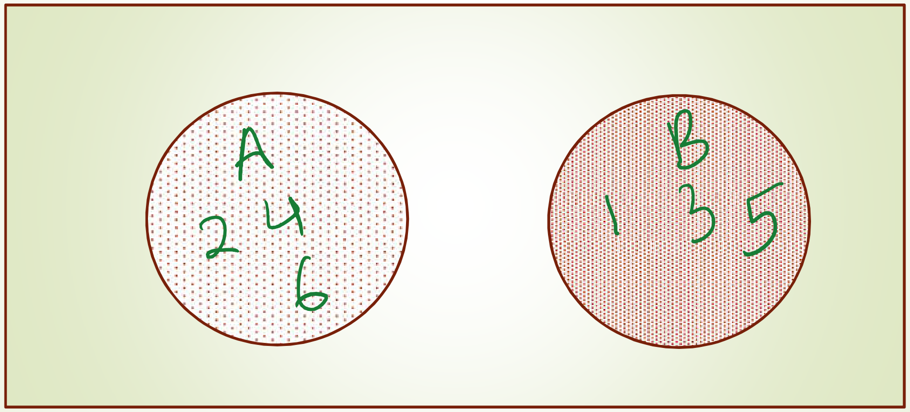
B –A = ??



Mutually exclusive events

- Events A and B are *mutually exclusive* if the occurrence of any one of them excludes the occurrence of the other event, i.e., if they can not occur simultaneously.

Example – Rolling a die. Event A is getting even no. & B is getting a odd no.



Exhaustive events

- Lots of events that together form a sample space.

Example – Rolling a die. Event A is getting even no. & B is getting a odd no.

A={2,4,6} and B={1,3,5}

A U B = {1,2,3,4,5,6} = **Sample space**

Example – A coin is tossed three times, consider the following events.
A: 'No head appears',

B: 'Exactly one head appears' and

C: 'At least two heads appear'.

Do they form a set of mutually exclusive and exhaustive events?

Axiomatic Approach to Probability

- It is another way of describing probability of an event.
- In this axioms or rules are depicted to assign probabilities.
- Let S be a sample space containing outcomes $\omega_1, \omega_2, \ldots, \omega_n$, i.e., $S = \{\omega_1, \omega_2, \ldots, \omega_n\}$

(i) For any event E,

$P(E) \geq 0$; $P(S)=1$

Also, $0 \leq P(\omega_i) \leq 1$ for each $\omega_i \in S$

(ii) $P(\omega_1) + P(\omega_2) + \ldots + P(\omega_n) = 1$

(iii) For any event A, $P(A) = \sum P(\omega_i)$, $\omega_i \in A$

(iv) $P(\varphi) = 0$

Example- Let S be a sample space containing outcomes $\omega_1, \omega_2, \ldots, \omega_n$, i.e., $S = \{\omega_1, \omega_2, \ldots, \omega_n\}$.Which of the following assignments of probabilities to each outcome are valid?

Outcomes	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
(a)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
(b)	1	0	0	0	0	0
(c)	$\frac{1}{8}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{4}$	$-\frac{1}{3}$
(d)	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{3}{2}$
(e)	0.1	0.2	0.3	0.4	0.5	0.6

Probabilities of equally likely outcomes

- Let all the outcomes are equally likely to occur, i.e., the chance of occurrence of each simple event must be same.
  i.e. $P(\omega_i) = p$, for all $\omega_i \in S$ where $0 \le p \le 1$

- $\sum P(\omega_i) = 1$ i.e., $p + p + \ldots + p$ (n times) $= 1$
  $np = 1$ i.e., => $\boldsymbol{p = 1/n}$

Probability of the event 'A or B'

- P(A or B)= P $(A \cup B)$; **not** $P(A)+P(B)$

Probability of event 'A and B'

- P(A and B)= P $(A \cap B)$; **not** $P(A) \cap P(B)$

Probability of event 'not A'

P(A')= 1- P(A)

$A' = U - A$

Example- One card is drawn from a well shuffled deck of 52 cards. If each outcome is equally likely, calculate the probability that the card will be

(i) a diamond

(ii) not an ace

(iii) a black card

(iv) not a diamond

(v) not a black card

Example-Two students Anil and Ashima appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that

(a) Both Anil and Ashima will not qualify the examination.

(b) Atleast one of them will not qualify the examination and

(c) Only one of them will qualify the examination